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PROBLEMS FOR JUNIORS

JP.541. If a, b, c sides in acute ΔABC with s - semiperimeter; r - inradii and $x, y, z \in (0, \frac{\pi}{2})$ are such that:

$$\cos x = \frac{a}{b+c}; \cos y = \frac{b}{c+a}; \cos z = \frac{c}{a+b}$$

then

$$\left(\tan^2 \frac{x}{2} + \tan^2 \frac{y}{2} + \tan^2 \frac{z}{2}\right) \tan^2 \frac{x}{2} \tan^2 \frac{y}{2} \tan^2 \frac{z}{2} = \frac{r^2}{s^2}$$

Proposed by Daniel Sitaru - Romania

JP.542. Solve for real numbers:

$$\begin{cases} \frac{6x+6y}{9+4xy} = z \\ \frac{6y+6z}{9+4yz} = x \\ \frac{6z+6x}{9+4zx} = y \end{cases}$$

Proposed by Daniel Sitaru - Romania

JP.543. Find $x, y, z > 0$ such that $x + y + z = 1$ and

$$\left(\frac{x^5}{yz+1} + \frac{y^5}{zx+1} + \frac{z^5}{xy+1}\right)\left(\frac{x^7}{yz+1} + \frac{y^7}{zx+1} + \frac{z^7}{xy+1}\right) = \frac{1}{72900}$$

Proposed by Daniel Sitaru - Romania

JP.544. If $x, y \in \mathbb{R}$ then:

$$\log(1+3\sin^2 x)\cdot\log(1+3\cos^2 x \sin^2 y)\cdot\log(1+3\cos^2 x \cos^2 y) \le \log^3 2$$

Proposed by Daniel Sitaru - Romania

JP.545. If $x, y, z > 0$ then:

$$\frac{x}{7x+5y+5z} + \frac{y}{5x+7y+5z} + \frac{z}{5x+5y+7z} \le \frac{3}{17}$$

Proposed by Daniel Sitaru - Romania

JP.546. In ΔABC the following relationship holds:

$$\sum \frac{\sin^4 \frac{A}{2} + \sin^4 \frac{B}{2}}{\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2}} \ge \frac{3}{4}$$

Proposed by Marin Chirciu - Romania

JP.547. In ΔABC the following relationship holds:

$$\sum \frac{\cos^4\frac{A}{2}+\cos^4\frac{B}{2}}{\cos^2\frac{A}{2}+\cos^2\frac{B}{2}} \geq \frac{9r}{2R}$$

Proposed by Marin Chirciu - Romania

JP.548. If $x,y,z\in[0,\infty)$ solve the system:

$$\begin{cases} x^2=y(y+z)\\ y^2=z(z+x)\end{cases}$$

Proposed by Cristian Miu - Romania

JP.549. Let ABC be a triangle with inradius r and circumradius R. Prove that:

$$\sum \frac{\sin^3 A+\sin^3 B}{\sin^5 A+\sin^5 B} \leq \left(\frac{R}{r}\right)^2$$

The sum is over all cyclic permutations of (A,B,C).

Proposed by George Apostolopoulos - Greece

JP.550. The non-coplanar points are given: A,B,C and D. If $k=$ the middle of the segment $[BD]$, $(KM=$ bisector $\widehat{AKB}$, $M\in(AB)$, $(KP=$ bisector $\widehat{AKD}$, $P\in(AD)$, and $N\in(AC)$, such that $\frac{AC}{AN}-\frac{BD}{2AK}=1$. Prove that:

$$AN\cdot NC+AM\cdot MB\geq 2PD\cdot(AN+AM-AP)$$

Proposed by Gheorghe Molea - Romania

JP.551. Find the real numbers x,y,z knowing that they meet the conditions:

$$x+y+z=1; xy+(x+y)(z+1)=\frac{4}{3}$$

Proposed by Gheorghe Molea - Romania

JP.552. Justify if exists non-zero natural numbers a,b,c,d, different in pairs, such that we have:

$$a(b+c-a)=b(a+c-b)=c(a+b-c)=\frac{a+b+c}{d}$$

Proposed by Gheorghe Molea - Romania

JP.553. Prove that for any $a,b,c\in\mathbb{R}$, we have the inequality:

$$\frac{a^3+a}{a^4+a^2+1}+\frac{b^3+b}{b^4+b^2+1}+\frac{c^3+c}{c^4+c^2+1}\leq 2$$

Proposed by Laura Molea and Ghoerghe Molea - Romania

JP.554. Solve for integers:

$$x(x-1)^2+y(y-1)^2=x(3x+7y)$$

Proposed by Laura Molea and Ghoerghe Molea - Romania

JP.555. In ΔABC we know: $m(\widehat{A})=90^\circ, AD\perp BC$, $D\in(BC), DE\perp AB, E\in(AB), DF\perp AC, F\in(AC)$, $BE=a, CF=b, BC=c, a,b,c>0$. Prove that $c\le 2\sqrt{a^2+b^2}$.

Proposed by Gheorghe Molea - Romania

PROBLEMS FOR SENIORS

SP.541. Find all continuous functions $f:\mathbb{R}\to\mathbb{R}$ such that:

$$f\left(\frac{x}{3}\right)-3f(x)=15x;(\forall)x\in\mathbb{R}$$

Proposed by Daniel Sitaru - Romania

SP.542. If $z_1,z_2,z_3\in\mathbb{C};|z_1|=|z_2|=|z_3|=1;z_1+z_2+z_3=1$ then find:

$$\Omega=\left(\sum_{k=1}^{n}z_i^3\right)\left(\sum_{i=1}^{n}z_i^5\right)\left(\sum_{i=1}^{n}z_i^7\right)$$

Proposed by Daniel Sitaru - Romania

SP.543. Let be $f:[0,1]\to[0,20]; f(x)=2\cdot 3^x+4\cdot 5^x-6$. Find:

$$\Omega=\int_0^{20}f^{-1}(x)dx$$

Proposed by Daniel Sitaru - Romania

SP.544. Find the maximum value of $n\in\mathbb{N}^*$ such that:

$$\sum_{k=1}^{n}\frac{1}{(k+1)\sqrt{k+1}+k\sqrt{k}}<\frac{31}{32}$$

Proposed by Daniel Sitaru - Romania

SP.545. Let a,b,c be positive real numbers such that $a=\max\{a,b,c\}$ and $a^2b^5c^5\ge 1$, and let

$$F(a,b,c)=\sqrt{\frac{ab+bc+ca}{3}}-\sqrt[3]{abc}$$

Prove that:

$$F(a,b,c)\ge F\left(\frac{1}{a},\frac{1}{b},\frac{1}{c}\right)$$

Proposed by Vasile Cîrtoaje and Vasile Mircea Popa - Romania

SP.546. If $a,b,c\in(0,1)$ then:

$$\sqrt{3a-a^2}+\sqrt{5b-b^2}+\sqrt{7c-c^2}\le\sqrt{15(a+b+c)-(a+b+c)^2}$$

Proposed by Daniel Sitaru - Romania

SP.547. Prove that if $x,y>1$ then:

$$\ln x\cdot\ln y(\sqrt[3]{\log_x y}+\sqrt[3]{\log_y x})^3\le 2\ln^2(xy)$$

Proposed by Daniel Sitaru - Romania

SP.548. If $a,b,c>0$ then:

$$(\sqrt{a}+\sqrt{b}+\sqrt{c})^2+2\left(\frac{ab}{a+b}+\frac{bc}{c+a}+\frac{ca}{a+b}\right)\ge 4(\sqrt{ab}+\sqrt{bc}+\sqrt{ca})$$

Proposed by Daniel Sitaru - Romania

SP.549. Let be ΔABC with sides a,b,c. Let be $x,y,z\in\mathbb{R}$ such that:

$$\cos x=\frac{a}{b+c};\cos y=\frac{b}{c+a};\cos z=\frac{c}{a+b}$$

Prove that:

$$\tan\frac{x}{2}\tan\frac{y}{2}\tan\frac{z}{2}\le\frac{\sqrt{3}}{9}$$

Proposed by Daniel Sitaru - Romania

SP.550. Prove that $(\forall)x\in(0,1)$ and $n\in\mathbb{N}^*$, we have the inequality:

$$x(1+x)(1-x^n)<\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{n+1}$$

Proposed by Gheorghe Molea - Romania

SP.551. Find:

$$\Omega=\lim_{n\to\infty}n\int_0^1\frac{x^3(1+x^{4n-8})}{(1+x^4)^n}dx$$

Proposed by Daniel Sitaru - Romania

SP.552. If $a>0$ then find:

$$\Omega=\int_{-a}^{a}\log_a(\sqrt{a^2x^2+1}-ax)dx$$

Proposed by Daniel Sitaru - Romania

SP.553. If $0 \le a \le b < 1$ then:

$$6\int_a^b \log\left(\frac{1+x}{1-x}\right)dx \ge (b^2-a^2)(b^2+a^2+6)$$

Proposed by Daniel Sitaru - Romania

SP.554. Find:

$$\Omega = \int \frac{8x-1}{e^{8x}+7x}dx; x \in (0,\infty)$$

Proposed by Daniel Sitaru - Romania

SP.555. Find:

$$\Omega = \lim_{x\to 0}\frac{1}{x}\left(\frac{1}{x}\int_0^x \frac{dt}{t+e^t} - 1\right)$$

Proposed by Daniel Sitaru - Romania

UNDERGRADUATE PROBLEMS

UP.541. Find:

$$\Omega = \int_1^{\sqrt{3}} \frac{x-\tan^{-1}x}{(1+x^2)^2(\tan^{-1}x)^3}dx$$

Proposed by Daniel Sitaru - Romania

UP.542. Find:

$$\Omega = \sum_{n=1}^{\infty}\tan^{-1}\left(\frac{49}{49+(7n+1)(7n+8)}\right)$$

Proposed by Daniel Sitaru - Romania

UP.543. Prove:

$$\int_0^1 \left(\frac{x^2\log(x)}{1+x^2}\right)^2 dx = G + 2 - \frac{3\pi^3}{32}$$

G represents the Catalan's constant.

Proposed by Said Attaoui - Algeria

UP.544. For $x, y, z > 0$ let us denote:

$$F(x,y,z) = \frac{xyz(xy+yz+zx)}{x^3y^3+y^3z^3+z^3x^3}[x^2(y-z)^2+y^2(z-x)^2+z^2(x-y)^2]$$

If $u, v, w \geq 1$, prove that:

$$F(u,v,w) \geq F\left(\frac{1}{u}, \frac{1}{v}, \frac{1}{w}\right)$$

Proposed by Vasile Mircea Popa - Romania

UP.545. Find:

$$\Omega = \int_0^{\frac{1}{2}} \frac{x^5-3x^3}{3x^6-x^4-3x^2+1}dx$$

Proposed by Daniel Sitaru - Romania

UP.546. Prove without any software:

$$\int_0^2 \sqrt[3]{\frac{x}{2}+\sqrt{\frac{1}{27}+\frac{x^2}{4}}}dx + \int_0^2 \sqrt[3]{\frac{x}{2}-\sqrt{\frac{1}{27}+\frac{x^2}{4}}}dx > \frac{5}{4}$$

Proposed by Daniel Sitaru - Romania

UP.547. Find without software:

$$\Omega = \int_1^2 \frac{4x^4-6x-9}{x^4}e^{2x+\frac{3}{x}}dx$$

Proposed by Daniel Sitaru - Romania

UP.548. Find:

$$\Omega = \int_1^e \frac{1-\ln x}{x^2+\ln^2 x}dx$$

Proposed by Daniel Sitaru - Romania

UP.549. If $0 < a \leq b$ then find:

$$\Omega(a,b) = \int_a^b \frac{\ln x}{x^2+(a+b)x+ab}dx$$

Proposed by Daniel Sitaru - Romania

UP.550. If $f : [0,1] \to \mathbb{R}; f$ continuous and

$$\int_0^1 xf(x)dx = a; \int_0^1 f(x)dx = b; a, b \in \mathbb{R}$$

then

$$\int_0^1 f^2(x)dx \geq 3(a-b)^2$$

Proposed by Daniel Sitaru - Romania

UP.551. Calculate the integral:

$$\int_0^\infty \frac{\arctan x}{\sqrt{3x^4+x^2+3}}dx$$

Proposed by Vasile Mircea Popa - Romania

UP.552. Calculate the integral:

$$\int_1^\infty \frac{\sqrt{x}\ln x}{(x+1)(x^2+1)}dx$$

Proposed by Vasile Mircea Popa - Romania

UP.553. Prove the equality:

$$\int_0^\infty \frac{|\cos(x)|}{1+x^2}dx = 1-2\sum_{n=1}^{\infty}\frac{(-1)^n}{(4n^2-1)e^{2n}}$$

Proposed by Vasile Mircea Popa - Romania

UP.554. We consider the equation: $(1+iz)^{2n} = i\cdot(1+z^2)^n$, where $n \geq 1$ natural number and $i^2=-1$.
a. Prove that the complex number i is a solution of the equation for any $n \geq 1$.
b. Solve the equation in the case $n=1$ and in one of the cases $n=2$ or $n=3$.
c. Find the solution of the equation in the general case $n \in \mathbb{N}^*$.

Proposed by Adalbert Kovacs - Romania

UP.555. Find the solution of the system:

$$\sqrt{8x+5}+\sqrt{9y+6} = \sqrt{8x+9y+29} \qquad \sqrt{12x+19}-\sqrt{3y+15} = \sqrt{12x+3y-6}$$

Proposed by Bela Kovacs - Romania

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