A NEW GENERALIZATION FOR TSINTSIFAS' INEQUALITY

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Abstract. In this paper we will give a new generalization for Tsintsifas' inequality and it consequence: Goldner's inequality.

Main result:

If $m \geq 0; M \in Int(\Delta ABC); x, y, z > 0; d_a = d(A, BC); d_b = d(B, AC)$, $d_c = d(C, AB)$ then in ΔABC the following relationship holds:

$$\sum_{cyc} \frac{x^{m+1}}{(y+z)^{m+1}} \cdot \frac{a^{3m+4}}{d_a^m} \geq 2^{2m+3} \cdot F^{m+2} \tag{1}$$

If $m = 0$ then (1) becomes the classical Tsintsifas' inequality:

$$\frac{x}{y+z} \cdot a^4 + \frac{y}{z+x} \cdot b^4 + \frac{z}{x+y} \cdot c^4 \geq 8F^2 \tag{2}$$

Proof.

Denote: $F_a = [MBC]; F_b = [MCA]; F_c = [MAB]$

$$\sum_{cyc} \frac{x^{m+1}}{(y+z)^{m+1}} \cdot \frac{a^{3m+4}}{d_a^m} = \sum_{cyc} \frac{x^{m+1}}{(y+z)^{m+1}} \cdot \frac{a^{4m+4}}{(ad_a)^m} =$$

$$= \sum_{cyc} \frac{(\frac{xa^4}{y+z})^{m+1}}{(2F_a)^m} \overset{\text{RADON}}{\geq} \frac{(\sum_{cyc} \frac{xa^4}{y+z})^{m+1}}{2^m(F_a+F_b+F_c)^m} =$$

$$= \frac{1}{2^m \cdot F^m} \cdot \left(\sum_{cyc} \frac{xa^4}{y+z} \right)^{m+1} \tag{3}$$

$$\sum_{cyc} \frac{xa^4}{y+z} = \sum_{cyc} \frac{x^2a^4}{xy+xz} \overset{\text{BERGSTRÖM}}{\geq}$$

$$\geq \frac{(xa^2+yb^2+zc^2)^2}{(xy+xz)+(yz+yx)+(zx+zy)} \overset{\text{KLAMKIN}}{\geq}$$

$$\geq \frac{16(xy+yz+zx)F^2}{2(xy+yz+zx)} = 8F^2 \tag{4}$$

By (3):

$$\sum_{cyc} \frac{x^{m+1}}{(y+z)^{m+1}} \cdot \frac{a^{3m+4}}{d_a^m} \geq \frac{1}{2^m \cdot F^m} \left(\sum_{cyc} \frac{xa^4}{y+z} \right)^{m+1} \geq$$

$$\overset{(4)}{\geq} \frac{1}{2^m \cdot F^m} \cdot (8F^2)^{m+1} = \frac{2^{3m+3} \cdot F^{2m+2}}{2^m \cdot F^m} = 2^{2m+3} \cdot F^{m+2}$$

Observation:
If we take in $(2) : x = y = z$ then we obtain Goldner's inequality:

$$\frac{1}{1+1} \cdot a^4 + \frac{1}{1+1} \cdot b^4 + \frac{1}{1+1} \cdot c^4 \geq 8F^2$$

$$\frac{1}{2}(a^4 + b^4 + c^4) \geq 8F^2$$

$$a^4 + b^4 + c^4 \geq 16F^2$$

Equality holds for: $a = b = c$. □

References

[1] Romanian Mathematical Magazine - Interactive Journal, www.ssmrmh.ro

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