ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c\in[0,2]$ then:

$$\frac{a^2+b^2+c^2}{3}\le\frac{(a+b+c)^2}{3}+1$$

Proposed by Daniel Sitaru – Romania

Solution 1 by Sakthi Vel-India

$$\frac{(a+b+c)^2}{3}+1-\frac{(a^2+b^2+c^2)}{3}\ge 0$$

$$\frac{a^2+b^2+c^2}{3}+\frac{2ab+2bc+2ac}{3}-\frac{(a^2+b^2+c^2)}{3}+1\ge$$

$$\ge\frac{6[abc]^{\frac{3}{6}}}{3}+1=2(abc)^{\frac{1}{2}}+1\ge 0$$

which is true since $a,b,c\in[0,2]$

Solution 2 by Fayssal Abdelli-Algeria

$$a,b,c\in[0,2]$$

$$\frac{a^2+b^2+c^2}{3}\overset{?}{\le}\frac{(a+b+c)^2}{3}+1$$

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ac)$$

$$\Leftrightarrow (a+b+c)^2\ge a^2+b^2+c^2 \text{ because } 2(ab+bc+ca)\ge 0$$

$$\Rightarrow\frac{(a+b+c)^2}{3}\ge\frac{(a^2+b^2+c^2)}{3}\Rightarrow 1+\frac{(a+b+c)^2}{3}\ge\frac{a^2+b^2+c^2}{3}$$