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If $a > \frac{1}{2}$, $b > \frac{1}{4}$, $c\frac{1}{3}$, prove the inequality

$$\frac{3a^2}{4b-1} + \frac{2b^2}{9c-3} + \frac{c^2}{2a-1} \geq 2.$$ **In what case is equality possible?**

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

Let $x := 3a + 2b + 3c$. By CBS inequality, we have

$$\frac{3a^2}{4b-1} + \frac{2b^2}{9c-3} + \frac{c^2}{2a-1} \geq \frac{(3a+2b+3c)^2}{3(4b-1)+2(9c-3)+9(2a-1)} =$$

$$= \frac{x^2}{6(x-3)} = 2 + \frac{(x-6)^2}{6(x-3)} \geq 2$$

Equality holds when $3a + 2b + 3c = x = 6$ and

$$\frac{a}{4b-1} = \frac{b}{9c-3} = \frac{c}{3(2a-1)} = \frac{3.a + 2.b + 3.c}{3.(4b-1) + 2.(9c-3) + 3.3(2a-1)} =$$

$$= \frac{x}{6x-18} = \frac{1}{3}$$ **then** $a = \frac{17}{21}, b = \frac{6}{7}, c = \frac{13}{21}$