ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0$ then:

$$\frac{a^3}{b}+\frac{b^3}{a}+8\left(\frac{1}{a+1}+\frac{1}{b+1}\right)\geq 10$$

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Solution by Tapas Das-India

$$a^4+b^4 \overset{CEBYSHEV}{\geq} \frac{(a^3+b^3)(a+b)}{2}=$$

$$\frac{(a+b)^2(a^2+b^2-ab)}{2}\overset{AM-GM}{\geq}\frac{(a+b)^2(2ab-ab)}{2}=\frac{ab(a+b)^2}{2}\ and$$

$$8\left(\frac{1}{a+1}+\frac{1}{b+1}\right)\overset{Bergstrom}{\geq} 8\cdot\frac{(1+1)^2}{a+b+2}=\frac{32}{a+b+2}$$

$$Now\ we\ need\ to\ show$$

$$\frac{a^3}{b}+\frac{b^3}{a}+8\left(\frac{1}{a+1}+\frac{1}{b+1}\right)\geq 10\ or$$

$$\frac{a^4+b^4}{ab}+8\left(\frac{1}{a+1}+\frac{1}{b+1}\right)\geq 10\ or$$

$$\frac{ab(a+b)^2}{2ab}+\frac{32}{a+b+2}\geq 10\ or$$

$$\frac{x^2}{2}+\frac{32}{x+2}\overset{a+b=x>0}{\geq} 10\ or$$

$$x^3+2x^2-20x+24\geq 0\ or$$

$$(x+6)(x-2)^2\geq 0\ true$$

$$Equality\ for\ x=a+b=2\ or\ a=b=1$$