ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0$ then:

$$\frac{a^3}{a^2+1}+\frac{b^3}{b^2+1}+\frac{4}{a+b}\geq 3$$

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Solution by Tapas Das-India

$$\frac{a^3}{a^2+1}+\frac{b^3}{b^2+1}+\frac{4}{a+b}\geq 3$$

$$\left(a-\frac{a}{a^2+1}\right)+\left(b-\frac{b}{b^2+1}\right)+\frac{4}{a+b}\geq 3$$

$$(a+b)+\frac{4}{a+b}-\left(\frac{a}{a^2+1}+\frac{b}{b^2+1}\right)\geq 3$$

$$(a+b)+\frac{4}{a+b}-\left(\frac{a}{2a}+\frac{b}{2b}\right)\geq 3(am-gm)$$

$$(a+b)+\frac{4}{a+b}\geq 4$$

$$(a+b)^2-4(a+b)+4\geq 0\ or\ (a+b-2)^2\geq 0\ (true)$$

$$Equality\ for\ a+b=2\ or\ a=b=1$$