ROMANIAN MATHEMATICAL MAGAZINE

If $a,b>0$ then:

$$\sqrt{3a^2+1}+\sqrt{3b^2+1}+6\left(\frac{1}{a+1}+\frac{1}{b+1}\right)\geq 10$$

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$$\sqrt{3a^2+1}+\sqrt{3b^2+1}=\sqrt{\left(\sqrt{3}a\right)^2+(1)^2}+\sqrt{\left(\sqrt{3}b\right)^2+(1)^2}\overset{Minkowski}{\geq}$$

$$\sqrt{\left(\sqrt{3}(a+b)\right)^2+(1+1)^2}=\sqrt{3(a+b)^2+4}\ and$$

$$6\left(\frac{1}{a+1}+\frac{1}{b+1}\right)\overset{Bergstrom}{\geq}6.\frac{(1+1)^2}{a+b+2}=\frac{24}{a+b+2}$$

$$Now\ we\ need\ to\ show\ \sqrt{3a^2+1}+\sqrt{3b^2+1}+6\left(\frac{1}{a+1}+\frac{1}{b+1}\right)\geq 10\ or$$

$$\sqrt{3(a+b)^2+4}+\frac{24}{a+b+2}\geq 10\ or,$$

$$\sqrt{3x^2+4}+\frac{24}{x+2}\overset{a+b=x>0}{\geq}10\ or,$$

$$\sqrt{3x^2+4}\geq 10-\frac{24}{x+2}\ or,$$

$$\sqrt{3x^2+4}\geq\frac{10x-4}{x+2}\ or,$$

$$(3x^2+4)(x+2)^2\geq(10x-4)^2\ or,$$

$$x^3+4x^2-28x+32\geq 0\ or,$$

$$(x-2)^2(x+8)\geq 0\ true\ ,equality\ for\ x=a+b=2\ or,a=b=1$$