ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, abc=1$ then:

$$(a+b+c)^2\left(\frac{1}{a^2+2}+\frac{1}{b^2+2}+\frac{1}{c^2+2}\right)\geq 9$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$(a+b+c)^2=\sum a^2+2\sum ab \overset{AM-GM}{\geq}$$

$$\geq \sum a^2+6\sqrt[3]{a^2b^2c^2}=\sum a^2+6(as\ abc=1)=(a^2+2)+(b^2+2)+(c^2+2)$$

$$(a+b+c)^2\left(\frac{1}{a^2+2}+\frac{1}{b^2+2}+\frac{1}{c^2+2}\right)\geq$$

$$\geq [\,(a^2+2)+(b^2+2)+(c^2+2)\,]\ \left(\frac{1}{a^2+2}+\frac{1}{b^2+2}+\frac{1}{c^2+2}\right)\overset{Cauchy-Schwarz}{\geq} 9$$

$$(Equality\ for\ a=b=c=1)$$