

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c \geq 0, a + b + c = 3$ then:

$$\frac{a}{b^2 + c^2 + 2a + 2} + \frac{b}{c^2 + a^2 + 2b + 2} + \frac{c}{a^2 + b^2 + 2c + 2} \leq \frac{1}{2}$$

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Solution by Tapas Das-India

$$\begin{aligned} \sum_{cyc} \frac{a}{b^2 + c^2 + 2a + 2} &= \sum_{cyc} \frac{a}{(b^2 + 1) + (c^2 + 1) + 2a} \leq \\ &\stackrel{AM-GM}{\lesssim} \sum_{cyc} \frac{a}{2a + 2b + 2c} = \frac{a+b+c}{2(a+b+c)} = \frac{1}{2} \end{aligned}$$

Equality for $a = b = c = 1$