ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\frac{a}{a+1}+\frac{b}{b+1}+\frac{16}{a+b+6}\le 3$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Khaled Abd Imouti-Syria

$$\frac{a}{a+1}+\frac{b}{b+1}+\frac{16}{a+b+6}\le 3\Leftrightarrow$$

$$\Leftrightarrow\frac{a}{a+1}-1+\frac{b}{b+1}-1+\frac{16}{a+b+6}-1\le 0\Leftrightarrow$$

$$\Leftrightarrow\frac{-1}{a+1}+\frac{-1}{b+1}+\frac{10-a-b}{a+b+6}\le 0\Leftrightarrow$$

$$\Leftrightarrow\frac{1}{a+1}+\frac{1}{b+1}\ge\frac{10-(a+b)}{(a+b)+6}\ (to\ prove)$$

$$\frac{1}{a+1}+\frac{1}{b+1}\overbrace{\ge}^{BERGSTROM}\frac{4}{a+b+2}$$

$$Denote\ a+b=x.\ \ Remains\ to\ prove:$$

$$\frac{4}{x+2}\ge\frac{10-x}{x+6}$$

$$4x+24\ge 10x-x^2+20-2x$$

$$x^2-4x+4\ge 0$$

$$(x-2)^2\ge 0$$

Equality holds for: $a=b=1$.