ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$ and $(x-1)^2 + \left(y - \frac{3}{2}\right)^2 + z^2 \le \frac{9}{4}$

$$\frac{x^3 + x^2 + 36}{2(x+1)} + \frac{y^3 + y^2 + 36}{4(y+1)} + \frac{2z^3 + z^2 + 9}{2z+1} \ge 16$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$4 \cdot \frac{x^3 + x^2 + 36}{2(x+1)} + 4 \cdot \frac{y^3 + y^2 + 36}{4(y+1)} + 4 \cdot \frac{2z^3 + z^2 + 9}{2z+1} \ge 64$$

$$\left(2x^2 + \frac{144}{2(x+1)}\right) + \left(y^2 + \frac{36}{y+1}\right) + \left(4z^2 + \frac{36}{2z+1}\right) \ge 64$$

LHS : $\frac{x^2}{1} + \frac{x^2}{1} + \frac{y^2}{1} + \frac{(2z)^2}{1} + \frac{12^2}{2(x+1)} + \frac{6^2}{y+1} + \frac{6^2}{2z+1} \overset{Begstrom}{\ge} \frac{(2x+y+2z+24)^2}{2x+2y+2z+8}$

Let's prove:

Let $2x + y + 2z = a > 0$ *.Then* $\frac{(a+24)^2}{a+8} \ge 64$

$$(a+24)^2 \ge 64(a+8) \Rightarrow a^2 + 48a + 576 - 64a - 512 \ge 0 \Rightarrow$$

$$\Rightarrow a^2 - 16a + 64 \ge 0 \Rightarrow (a-8)^2 \ge 0 \ \ (proved)$$

Equality holds if $x = 2, y = 2, z = 1 \Rightarrow (2; 2; 1)$