ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\ a+b+c=a^2b^2+b^2c^2+c^2a^2$ then:

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\geq 3$$

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Solution by Tapas Das-India

$$\forall\, x,y,z>0\,,\sum x^2y^2\geq xyz(x+y+z)$$

$$a+b+c=a^2b^2+b^2c^2+c^2a^2\geq abc(a+b+c)or\ abc\leq 1$$

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\overset{Am-Gm}{\geq}3\left(\frac{1}{abc}\right)^{\frac{2}{3}}\geq 3(since\ abc\leq 1)$$

$$Equality\ holds\ for\ a=b=c=1$$