ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\ a^2+b^2+c^2=1$ then:

$$\frac{a^5-2a^3+a}{b^2+c^2}+\frac{b^5-2b^3+b}{c^2+a^2}+\frac{c^5-2c^3+c}{a^2+b^2}\le\frac{2\sqrt{3}}{3}$$

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Solution by Tapas Das-India

$$(a^2+b^2+c^2)=1$$

By Cebyshev: $a^3+b^3+c^3\ge\frac{1}{3}(a^2+b^2+c^2)(a+b+c)=\frac{1}{3}(a+b+c)\ (1)$

By CBS: $(a+b+c)=\sqrt{(a+b+c)^2}\le\sqrt{3(a^2+b^2+c^2)}=\sqrt{3}\ \ (2)$

$$\frac{a^5-2a^3+a}{b^2+c^2}+\frac{b^5-2b^3+b}{c^2+a^2}+\frac{c^5-2c^3+c}{a^2+b^2}=$$

$$=\sum\frac{a^5-2a^3+a}{b^2+c^2}=\sum\frac{a(a^4-2a^2+1)}{1-a^2}=$$

$$=\sum\frac{a(a^2-1)^2}{1-a^2}=\sum a(1-a^2)=(a+b+c)-(a^3+b^3+c^3)\overset{(1)}{\le}$$

$$\le\sum a-\frac{1}{3}(a+b+c)=\frac{2}{3}\sum a\overset{(2)}{\le}\le\frac{2\sqrt{3}}{3}$$

$$Equality\ a=b=c=\frac{1}{\sqrt{3}}.$$