

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0$ then:

$$\frac{a^2}{b} + \frac{b^2}{a} + 4 \left(\frac{1}{a+1} + \frac{1}{b+1} \right) \geq 6$$

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Solution by Tapas Das-India

We need to show:

$$\frac{a^2}{b} + \frac{b^2}{a} + 4 \left(\frac{1}{a+1} + \frac{1}{b+1} \right) \geq 6 \text{ or,}$$

$$\frac{(a+b)^2}{a+b} + \frac{4(1+1)^2}{a+b+2} \geq 6 \text{ (Bergstrom)}$$

$$\text{or } x + \frac{16}{x+2} \stackrel{a+b=x>0}{\geq} 6 \text{ or}$$

$$16 \geq (6-x)(x+2) \text{ or,}$$

$$16 \geq 4x + 12 - x^2 \text{ or,}$$

$$x^2 - 4x + 4 \geq 0 \text{ or}$$

$$(x-2)^2 \geq 0 \text{ true.}$$

Equality for $x = a + b = 2$ or $a = b = 1$