ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{a^2+2b^2}{a^3+2b^3}+\frac{b^2+2c^2}{b^3+2c^3}+\frac{c^2+2a^2}{c^3+2a^3}$$

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Solution by Tapas Das-India

$$a^3+2b^3=a^3+b^3+b^3 \geq \frac{1}{3}(a+b+b)(a^2+b^2+b^2)=$$

$$=\frac{1}{3}(a+b+b)(a^2+2b^2)\ (1)$$

$$\frac{a^2+2b^2}{a^3+2b^3}+\frac{b^2+2c^2}{b^3+2c^3}+\frac{c^2+2a^2}{c^3+2a^3}=\sum\frac{a^2+2b^2}{a^3+2b^3}\overset{(1)}{\leq}$$

$$\leq 3\sum\frac{1}{a+b+b}\overset{AM-HM}{\leq}\frac{1}{3}\sum\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{b}\right)=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

$$Equality\ for\ a=b=c=1$$