

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c \geq 0$, $a^2 + b^2 + c^2 = 1$ then:

$$\frac{1}{a^3 + 2} + \frac{1}{b^3 + 2} + \frac{1}{c^3 + 2} \geq \frac{4}{3}$$

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Solution by Tapas Das-India

$$\begin{aligned} \frac{1}{a^3 + 2} + \frac{1}{b^3 + 2} + \frac{1}{c^3 + 2} &= \sum \frac{1}{a^3 + 2} = \\ &= \frac{1}{2} \sum \frac{2}{a^3 + 2} = \frac{1}{2} \sum \left(1 - \frac{a^3}{a^3 + 1 + 1} \right) \stackrel{AM-GM}{\geq} \frac{3}{2} - \frac{1}{2} \sum \frac{a^3}{3a} = \\ &= \frac{3}{2} - \frac{1}{6} \sum a^2 = \frac{3}{2} - \frac{1}{6} = \frac{4}{3} \end{aligned}$$

Equality for $(a, b, c) = (1, 0, 0)$ or $(0, 1, 0)$ or $(0, 0, 1)$