ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0, \frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=1$ then:

$$xyz(x+y+z)(x+1)(y+1)(z+1) \geq 1296$$

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Solution by Tapas Das-India

$$\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}=1 \quad (1)\ or$$

$$x+y+z+2=xyz\ (2)$$

$$[(x+1)+(y+1)+(z+1)]\left[\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}\right] \overset{C-S}{\geq} 9\ or$$

$$(x+y+z+3) \overset{(1)}{\geq} 9\ or\ x+y+z \geq 6\ (2)$$

$$Now\ from (2)\ xyz \geq 6+2=8\ \ (3)\ and$$

$$xy+yz+zx \overset{AM-GM}{\geq} 3(xyz)^{\frac{2}{3}} \overset{(3)}{\geq} 3.(8)^{\frac{2}{3}}=12\ \ (4)$$

$$xyz(x+y+z)(x+1)(y+1)(z+1)=$$
$$=xyz(x+y+z)(1+x+y+z+xy+yz+zx+xyz) \overset{(2)\&(3)\&(4)}{\geq}$$

$$\geq 8.6(1+6+12+8)=1296$$

$$Equality\ for\ \ x=y=z=2$$