ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0, y\in(0,z)\cup[x,\infty), x\ge z$ then:

$$\frac{xz}{y^2+yz}+\frac{y^2}{x^2+yz}+\frac{x+2z}{x+z}\ge\frac{5}{2}$$

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$$\frac{xz}{y^2+yz}+\frac{y^2}{xz+yz}+\frac{yz}{xz+y^2}\overset{NESBITT}{\ge}\frac{3}{2}$$

$$\frac{xz}{y^2+yz}+\frac{y^2}{xz+yz}+\frac{yz}{xz+y^2}+\frac{x+2z}{x+z}\ge\frac{3}{2}+\frac{x+2z}{x+z}$$

$$\frac{xz}{y^2+yz}+\frac{y^2}{xz+yz}+\frac{x+2z}{x+z}\ge\frac{3}{2}+1+\frac{z}{x+z}-\frac{yz}{xz+y^2}$$

$$\frac{xz}{y^2+yz}+\frac{y^2}{xz+yz}+\frac{x+2z}{x+z}\ge\frac{5}{2}+\frac{z}{x+z}-\frac{yz}{xz+y^2}$$

Let us prove:

$$\frac{z}{x+z}-\frac{yz}{xz+y^2}\ge 0\,(z\le x)(*)$$

$$\frac{1}{x+y}-\frac{y}{xz+y^2}\ge 0$$

$$y^2-(x+z)\mathrm{y}+\mathrm{xz}\ge 0$$

y	0		z		x		$+\infty$
$y^2-(x+z)\mathrm{y}+\mathrm{xz}$	||	+	0	−	0	+	

Inequality $(*)$ is true