

ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0, y \in (0, z) \cup [x, \infty), x \geq z$ then:

$$\frac{xz}{y^2 + yz} + \frac{y^2}{x^2 + yz} + \frac{x + 2z}{x + z} \geq \frac{5}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Khaled Abd Imouti-Damascus-Syria

$$\frac{xz}{y^2 + yz} + \frac{y^2}{xz + yz} + \frac{yz}{xz + y^2} \stackrel{\text{NESBITT}}{\geq} \frac{3}{2}$$

$$\frac{xz}{y^2 + yz} + \frac{y^2}{xz + yz} + \frac{yz}{xz + y^2} + \frac{x + 2z}{x + z} \geq \frac{3}{2} + \frac{x + 2z}{x + z}$$

$$\frac{xz}{y^2 + yz} + \frac{y^2}{xz + yz} + \frac{x + 2z}{x + z} \geq \frac{3}{2} + 1 + \frac{z}{x + z} - \frac{yz}{xz + y^2}$$

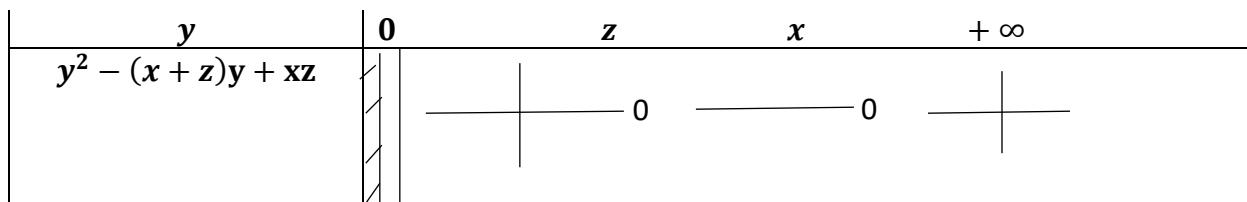
$$\frac{xz}{y^2 + yz} + \frac{y^2}{xz + yz} + \frac{x + 2z}{x + z} \geq \frac{5}{2} + \frac{z}{x + z} - \frac{yz}{xz + y^2}$$

Let us prove:

$$\frac{z}{x+z} - \frac{yz}{xz+y^2} \geq 0 \quad (z \leq x) (*)$$

$$\frac{1}{x+y} - \frac{y}{xz+y^2} \geq 0$$

$$y^2 - (x+z)y + xz \geq 0$$



Inequality () is true*