ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ and $abc(a+2)(b+2)(c+2)=27$ then:

$$2^a+2^b+2^c\geq 6$$

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$$abc(a+2)(b+2)(c+2)=27$$

$$\frac{(a+b+c)^3}{27}\cdot\frac{\left((a+2)+(b+2)+(c+2)\right)^3}{27}\overset{Am-Gm}{\geq}$$
$$\geq\ abc(a+2)(b+2)(c+2)=27\ or$$

$$x^3(x+6)^3\overset{x=a+b+c}{\geq}(27)^3\ or$$

$$x(x+6)\geq 27\ or\ x^2+6x\geq 27\ or$$

$$(x+3)^2\geq 36\ or\ x+3\geq 6$$

$$or\ x\geq 3\ (1)$$

$$2^a+2^b+2^c\overset{Am-Gm}{\geq}3\left(2^{a+b+c}\right)^{\frac{1}{3}}\overset{(1)}{\geq}3(2^3)^{\frac{1}{3}}=6$$

$$Equality\ for\ a=b=c=1$$