ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c \geq 0$, $a^2+b^2+c^2=1$ then:

$$\frac{1}{a^3+2}+\frac{1}{b^3+2}+\frac{1}{c^3+2} \geq \frac{4}{3}$$

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Solution by Tapas Das-India

$$\frac{1}{a^3+2}+\frac{1}{b^3+2}+\frac{1}{c^3+2}=\sum\frac{1}{a^3+2}=\frac{1}{2}\sum\frac{2}{a^3+2}=$$

$$=\frac{1}{2}\sum\left(1-\frac{a^3}{a^3+1+1}\right)\overset{AM-GM}{\geq}\frac{3}{2}-\frac{1}{2}\sum\frac{a^3}{3a}=$$

$$=\frac{3}{2}-\frac{1}{6}\sum a^2=\frac{3}{2}-\frac{1}{6}=\frac{4}{3}$$

$$Equality\ for\ (a,b,c)=(1,0,0)or(0,1,0)or(0,0,1)$$