ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\frac{1}{\sqrt{a+8}}+\frac{1}{\sqrt{b+8}}+\frac{1}{\sqrt{c+8}}=$ then:

$$2^a+2^b+2^c\geq 6$$

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Solution by Tapas Das-India

$$1=\frac{1}{\sqrt{a+8}}+\frac{1}{\sqrt{b+8}}+\frac{1}{\sqrt{c+8}}=$$

$$=\frac{1^{\frac{3}{2}}}{\sqrt{a+8}}+\frac{1^{\frac{3}{2}}}{\sqrt{b+8}}+\frac{1^{\frac{3}{2}}}{\sqrt{c+8}}\overset{Radon}{\geq}\frac{(3)^{\frac{3}{2}}}{(a+b+c+24)^{\frac{1}{2}}}$$

$$(a+b+c+24)^{\frac{1}{2}}\geq 3^{\frac{3}{2}}$$

$$a+b+c+24\geq 27$$

$$a+b+c\geq 3$$

$$2^a+2^b+2^c\overset{AM-GM}{\geq}3\left(2^{a+b+c}\right)^{\frac{1}{3}}\geq 3(2^3)^{\frac{1}{3}}=6$$

$$Equality\ for\ a=b=c=1$$