ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\leq 2$ then:

$$\frac{1}{\sqrt{5a^2+2ab+2b^2}}+\frac{1}{\sqrt{5b^2+2bc+2c^2}}+\frac{1}{\sqrt{5c^2+2ca+2a^2}}\leq\frac{2}{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$5a^2+2ab+2b^2=4a^2+2ab+b^2+(a^2+b^2)\overset{AM-GM}{\geq}$$

$$\geq 4a^2+4ab+b^2=(2a+b)^2$$

$$LHS\leq\sum\frac{1}{2a+b}=\sum\frac{1}{a+a+b}\overset{AM-HM}{\leq}$$

$$\leq\frac{1}{9}\sum\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{3}{9}\sum(\frac{1}{a}+\frac{1}{b}+\frac{1}{c})\leq\frac{2}{3}$$

Equality holds for: $a=b=c=\frac{3}{2}$.