

ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$ then:

$$\frac{2x}{x^6 + y^4} + \frac{2y}{y^6 + z^4} + \frac{2z}{z^6 + x^4} \leq \frac{1}{x^4} + \frac{1}{y^4} + \frac{1}{z^4}$$

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Solution by Tapas Das-India

$$\begin{aligned} \frac{2x}{x^6 + y^4} + \frac{2y}{y^6 + z^4} + \frac{2z}{z^6 + x^4} &\stackrel{AM-GM}{\leq} \sum \frac{2x}{2x^3y^2} = \sum \frac{1}{x^2y^2} = \frac{x^2 + y^2 + z^2}{x^2y^2z^2} = \\ &= \frac{x^2y^2z^2(x^2 + y^2 + z^2)}{x^4y^4z^4} = \frac{\sum(x^2y^2)(x^2z^2)}{x^4y^4z^4} \stackrel{(\sum ab) \leq (\sum a^2) \forall a,b,c>0}{\leq} \\ &\leq \frac{x^4y^4 + y^4z^4 + z^4x^4}{x^4y^4z^4} = \frac{1}{x^4} + \frac{1}{y^4} + \frac{1}{z^4} \end{aligned}$$

Equality holds for $x = y = z = 1$.