ROMANIAN MATHEMATICAL MAGAZINE

$$If\ a,b,c>0\ then:$$

$$\frac{b+c}{a+\sqrt[3]{4b^3+4c^3}}+\frac{c+a}{b+\sqrt[3]{4c^3+4a^3}}+\frac{a+b}{c+\sqrt[3]{4a^3+4b^3}}\le 2$$

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Solution by Tapas Das-India

$$\forall x,y>0\ \frac{x^3+y^3}{2}\ge\left(\frac{x+y}{2}\right)^3\ or,4x^3+4y^3\ge(x+y)^3\ \ (1)$$

$$\frac{b+c}{a+\sqrt[3]{4b^3+4c^3}}+\frac{c+a}{b+\sqrt[3]{4c^3+4a^3}}+\frac{a+b}{c+\sqrt[3]{4a^3+4b^3}}=$$

$$=\sum.\frac{b+c}{a+\sqrt[3]{4b^3+4c^3}}\overset{(1)}{\le}\sum\frac{b+c}{a+\sqrt[3]{(b+c)^3}}=$$

$$=\sum\frac{b+c}{a+b+c}=\frac{2(a+b+c)}{a+b+c}=2$$

$$Equality\ for\ a=b=c$$