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If $a, b, c > 0, \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = 3$ then:

$$2^a + 2^b + 2^c \geq 6$$

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Solution by Daniel Sitaru-Romania

$$2^a + 2^b + 2^c \overset{AM-GM}{\geq} 3 \cdot \sqrt[3]{2^a \cdot 2^b \cdot 2^c} = 3\sqrt[3]{2^{a+b+c}} \geq$$

$$\overset{AM-HM}{\geq} 3 \cdot \sqrt[3]{2^{\frac{9}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}}} = 3 \cdot \sqrt[3]{2^{\frac{9}{3}}} = 3 \cdot \sqrt[3]{2^3} = 6$$

Equality holds for: $a = b = c = 1$.