ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ and $a^3+b^3+c^3=3$ then:

$$\frac{a^3}{b^2-2b+3}.+\frac{2b^3}{c^3+a^2-2a-3c+7}+\frac{3c^3}{a^4+b^4+a^2-2b^2-6a+11}\le\frac{3}{2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$b^2-2b+3=(b^2+1)-2b+2\overset{AM-GM}{\ge}2b-2b+2=2\ (1)$$

$$c^3+a^2-2a-3c+7=(c^3+1+1)+(a^2+1)-2a-3c+4\overset{AM-GM}{\ge}$$

$$\ge 3c+2a-2a-3c+4=4\ \ (2)$$

$$a^4+b^4+a^2-2b^2-6a+11=$$
$$=(a^4+a^2+1+1+1+1)+(b^4+1)-2b^2-6a+6\overset{AM-GM}{\ge}$$

$$\ge 6a+2b^2-2b^2-6a+6=6\ (3)$$

$$\frac{a^3}{b^2-2b+3}.+\frac{2b^3}{c^3+a^2-2a-3c+7}+\frac{3c^3}{a^4+b^4+a^2-2b^2-6a+11}\overset{(1),(2),(3)}{\le}$$

$$\le\frac{a^3}{2}+\frac{2b^3}{4}+\frac{3c^3}{6}=\frac{1}{2}(a^3+b^3+c^3)=\frac{3}{2}$$

Equality for $a=b=c=1$