ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0$ then:

$$\frac{a^2+b^2}{ab}+\frac{8\sqrt{ab}}{a+b}\geq 6$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$a+b\overset{AM-GM}{\geq}2\sqrt{ab}\ or\ m\overset{m=a+b,ab=u^2}{\geq}2u\ (1)$$

$$\frac{a^2+b^2}{ab}+\frac{8\sqrt{ab}}{a+b}\geq 6\ or$$

$$\frac{(a+b)^2-2ab}{ab}+\frac{8\sqrt{ab}}{a+b}\geq 6\ or$$

$$\frac{(a+b)^2}{ab}-2+\frac{8\sqrt{ab}}{a+b}\geq 6$$

$$or\ \frac{(a+b)^2}{ab}+\frac{8\sqrt{ab}}{a+b}\geq 8\ or$$

$$\frac{m^2}{u^2}+\frac{8m}{m}\geq 8\ (\ m=a+b, ab=u^2\ )$$

$$or\ m^3-8mu^2+8u^3\geq 0\ or$$

$$(m-2u)(m^2-4u^2+2mu)\geq 0\ true\ (as\ \ m\geq 2u)$$

$$Equality\ for\ a=b=1$$