

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0, a + b + c = 3$ then:

$$\frac{a^3}{9 - a^2} + \frac{b^3}{9 - b^2} + \frac{c^3}{9 - c^2} \geq \frac{3}{8}$$

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Lemma : For $x > 0$ we have:

$$\frac{x^3}{9 - x^2} \geq \frac{13x - 9}{32}$$

Proof:

$$\frac{x^3}{9 - x^2} \geq \frac{13x - 9}{32} \text{ or}$$

$$45x^3 - 9x^2 - 117x + 81 \geq 0 \text{ or}$$

$$(x - 1)^2(45x + 81) \geq 0 \text{ (true as } x > 0)$$

$$\sum \frac{a^3}{9 - a^2} \geq \sum \frac{(13a - 9)}{32} = \frac{(13\sum a - 27)}{32} = \frac{3}{8} \text{ (as } a + b + c = 3)$$