ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, abc=1$ then:

$$\sum\frac{a(b^3+c^3)}{(a+b)(a+c)}\ge\frac{3}{2}$$

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$$(1^3+1^3)(1^3+1^3)(b^3+c^3)\overset{HUYGENS}{\geq}(b+c)^3$$

$$(b^3+c^3)\ge\frac{(b+c)^3}{4}\ (1)$$

$$\prod(a+b)\overset{Cesaro}{\geq}8abc\ (2)$$

$$\sum\frac{a(b^3+c^3)}{(a+b)(a+c)}\overset{(1)}{\geq}\sum\frac{a(b+c)^3}{4(a+b)(a+c)}\overset{AM-GM}{\geq}$$

$$\ge\frac{3}{4}\sqrt[3]{abc(a+b)(b+c)(c+a)}\overset{(2)}{\geq}\frac{3}{4}\sqrt[3]{8a^2b^2c^2}=\frac{3}{2}\ (as\ \ abc=1)$$

$$Equality\ holds\ \ for\ \ a=b=c=1$$