ROMANIAN MATHEMATICAL MAGAZINE

SP.554 Find:

$$\Omega = \int \frac{8x-1}{e^{8x}+7x}dx; x \in (0, \infty)$$

Proposed by Daniel Sitaru – Romania

Solution 1 by proposer

$$\Omega = \int \frac{8x-1}{e^{8x}+7x}dx = \frac{1}{7}\int \frac{7(8x-1)}{e^{8x}+7x}dx =$$

$$= \frac{1}{7}\int \frac{56x-7}{e^{8x}+7x}dx = \frac{1}{7}\int \frac{8e^{8x}+56x-8e^{8x}-7}{e^{8x}+7x}dx =$$

$$= \frac{1}{7}\int \frac{8(e^{8x}+7x)}{e^{8x}+8x}dx - \frac{1}{7}\int \frac{8e^{8x}+7}{e^{8x}+7x}dx =$$

$$= \frac{1}{7}\int 8\,dx - \frac{1}{7}\int \frac{(e^{8x}+7x)'}{e^{8x}+7x}dx = \frac{8x}{7} - \frac{1}{7}\ln(e^{8x}+7x) + C$$

Solution 2 by Marin Chirciu-Romania

$$\Omega = \int \frac{8x-1}{e^{8x}+7x}dx = \int \left(\frac{8}{7} - \frac{1}{7}\cdot\frac{8e^{8x}+7}{e^{8x}+7x}\right) = \frac{8}{7}x - \frac{1}{7}\ln(e^{8x}+8x) + C$$

Remark:

If $m \in \mathbb{R}, n \in (0, \infty)$ then find:

$$\Omega = \int \frac{mx-1}{e^{mx}+nx}dx.$$

Solution.

$$\Omega = \int \frac{mx-1}{e^{mx}+nx}dx = \int \left(\frac{m}{n} - \frac{1}{n}\cdot\frac{me^{mx}+n}{e^{mx}+nx}\right) = \frac{m}{n}x - \frac{1}{n}\ln(e^{mx}+nx) + C$$