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J.2344 If $x, y > 0$, then in ABC the following relationship holds:

$$\frac{a^3}{xr+yh_a}+\frac{b^3}{xr+yh_b}+\frac{c^3}{xr+yh_c}\ge\frac{24}{x+3y}F$$

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Solution by Titu Zvonaru-Romania

We have $ah_a = bh_b = ch_c = 2F$ and $ar + br + cr = 2sr = 2F$.

Applying Bergström inequality and Ionescu-Weitzenbock inequality $a^2 + b^2 + c^2 \ge 4\sqrt{3}F$, we obtain:

$$\frac{a^3}{xr+yh_a}+\frac{b^3}{xr+yh_b}+\frac{c^3}{xr+yh_c}=$$

$$=\frac{a^4}{xar+yah_a}+\frac{b^4}{xbr+ybh_b}+\frac{c^4}{xcr+ych_c}\ge$$

$$\overset{BERGSTROM}{\overbrace{\ge}}\frac{(a^2+b^2+c^2)^2}{x(ar+br+cr)+y(ah_a+bh_b+ch_c)}=\frac{(a^2+b^2+c^2)^2}{2F(x+3y)}\ge$$

$$\overset{IONESCU-WEITZENBOCK}{\overbrace{\ge}}\frac{\left(4\sqrt{3}F\right)^2}{2F(x+3y)}=\frac{24}{x+3y}F.$$

Equality holds if and only if ΔABC is equilateral.