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J.2346 If $m \geq 0, x, y > 0$, then

$$(x^{2m+2}+1)(y^{2m+2}+1) \geq \frac{(x+y)^{2m+2}}{4^m}$$

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Applying Hölder inequality we obtain

$$\underbrace{(1+1)(1+1)\dots(1+1)}_{2m}(x^{2m+2}+1)(1+y^{2m+2}) \geq$$

$$\geq \left(\left(x^{2m+2}\cdot\underbrace{1\cdot 1\cdot \dots \cdot 1}_{2m}\right)^{\frac{1}{2m+2}} + \left(y^{2m+2}\cdot\underbrace{1\cdot 1\cdot \dots \cdot 1}_{2m}\right)^{\frac{1}{2m+2}}\right)^{2m+2} = (x+y)^{2m+2}.$$

It results that

$$2^{2m}(x^{2m+2}+1)(1+y^{2m+2}) \geq (x+y)^{2m+2},$$

hence

$$(x^{2m+2}+1)(y^{2m+2}+1) \geq \frac{(x+y)^{2m+2}}{4^m}.$$

Equality holds if and only if $x = y = 1$.