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J.2347 If $x, y > 0$, then in triangle ABC holds:

$$(x^2+y^2)(r_a^2+r_b^2+r_c^2) \geq 6xy\sqrt{3}F + (xr_a - yr_b)^2 + (xr_b - yr_c)^2 + (xr_c - yr_a)^2$$

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Solution by Titu Zvonaru-Romania

After some calculations the inequality is equivalent to

$$xy(r_ar_b + r_br_c + r_cr_a) \geq 3xy\sqrt{3}F \quad (1)$$

By the items 5.36 and 4.5 from $[1]$ we have

$$4(r_ar_b + r_br_c + r_cr_a) \geq 3(ab+bc+ca) \text{ and}$$

$ab+bc+ca \geq 4\sqrt{3}F$, hence $4(r_ar_b + r_br_c + r_cr_a) \geq 3(ab+bc+ca) \geq 12\sqrt{3}F$ and the inequality (1) is true.

Equality holds if and only if the triangle ABC is equilateral.

$[1]$ **O. Bottema, *Geometric Inequalities*, Groningen 1969**