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J.2349 If $M \in Int(\Delta ABC), d_a = d(M, BC), d_b = d(M, CA), d_c = d(M, AB)$, then:

$$\frac{a^{m+2}x^{m+1}}{d_a^m} + \frac{b^{m+2}y^{m+1}}{d_b^m} + \frac{c^{m+2}z^{m+1}}{d_c^m} \ge 2^{m+2}(xy + yz + zx)^{\frac{m+1}{2}} \cdot F$$

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We have $ad_a + bd_b + cd_c = 2F$.

Applying Radon's inequality and Oppenheim's inequality:

$a^2x + b^2y + c^2z \ge 4F\sqrt{xy + yz + zx}$ it follows that:

$$\frac{a^{m+2}x^{m+1}}{d_a^m} + \frac{b^{m+2}y^{m+1}}{d_b^m} + \frac{c^{m+2}z^{m+1}}{d_c^m} =$$

$$= \frac{a^{2m+2}x^{m+1}}{a^m d_a^m} + \frac{b^{2m+2}y^{m+1}}{b^m d_b^m} + \frac{c^{2m+2}z^{m+1}}{c^m d_c^m} =$$

$$= \frac{(a^2x)^{m+1}}{(ad_a)^m} + \frac{(b^2y)^{m+1}}{(bd_b)^m} + \frac{(c^2z)^{m+1}}{(cd_c)^m} \overset{RADON}{\ge} \frac{(a^2x + b^2y + c^2z)^{m+1}}{(ad_a + bd_b + cd_c)^m} \ge$$

$$\overset{OPPENHEIM}{\ge} \frac{\left(4F\sqrt{xy + yz + zx}\right)^{m+1}}{F^m} = 2^{m+2}(xy + yz + zx)^{\frac{m+1}{2}} \cdot F.$$

Equality holds if and only if ΔABC is equilateral and $x = y = z$.