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J.2356 If $m\geq 0$ and $M\in Int(\Delta ABC), d_a=d(M,BC), d_b=d(M,CA), d_c=d(M,AB)$, then

$$\frac{a^{m+1}b}{(xr+yd_b)^m}+\frac{b^{m+1}c}{(xr+yd_c)^m}+\frac{c^{m+1}a}{(xr+yd_a)^m}\geq\frac{2^{m+2}\left(\sqrt{3}\right)^{m+1}}{(x+y)^m}\cdot F.$$

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Solution by Titu Zvonaru-Romania

We have $(a+b+c)r=2F, ad_a+bd_b+cd_c=2F.$

Applying Radon's inequality and Gordon's inequality $ab+bc+ca\geq 4\sqrt{3}F$

it follows that:

$$\frac{a^{m+1}b}{(xr+yd_b)^m}+\frac{b^{m+1}c}{(xr+yd_c)^m}+\frac{c^{m+1}a}{(xr+yd_a)^m}=$$

$$=\frac{a^{m+1}b^{m+1}}{(xbr+ybd_b)^m}+\frac{b^{m+1}c^{m+1}}{(xcr+ycd_c)^m}+\frac{c^{m+1}a^{m+1}}{(xar+yad_a)^m}\geq\frac{(ab+bc+ca)^{m+1}}{(2xF+2yF)^m}\geq$$

$$\geq\frac{\left(4\sqrt{3}F\right)^{m+1}}{(x+y)^m(2F)^m}\cdot F=\frac{2^{m+2}\left(\sqrt{3}\right)^{m+1}}{(x+y)^m}\cdot F.$$

Equality holds if and only if ΔABC is equilateral.