J.2402 If $m \geq 0$, then in ΔABC holds:

$$\frac{a^2 \sin^{2m} A}{(\sin B \sin C)^m} + \frac{b^2 \sin^{2m} B}{(\sin C \sin A)^m} + \frac{c^2 \sin^{2m} C}{(\sin A \sin B)^m} \geq 36r^2$$

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Applying $AM - GM$ inequality, Carlitz's inequality $(abc)^{2/3} \geq \frac{4}{3}\sqrt{3}F$

and inequality $s^2 \geq 27r^2$ (item 5.11 from $[1]$), it follows that

$$\frac{a^2 \sin^{2m} A}{(\sin B \sin C)^m} + \frac{b^2 \sin^{2m} B}{(\sin C \sin A)^m} + \frac{c^2 \sin^{2m} C}{(\sin A \sin B)^m} \geq 3(abc)^{\frac{2}{3}} \geq$$

$$\geq 4\sqrt{3}F = 4\sqrt{3}rs \geq 4\sqrt{3}r\left(3\sqrt{3}r\right) = 36r^2.$$

Equality holds if and only if ΔABC is equilateral.

$[1]$ O. Bottema *Geometric Inequalities*, Groningen 1969