ROMANIAN MATHEMATICAL MAGAZINE

J.2405 Prove the cryptarithm:

$$ACDEA \times BCDEB \leq ACDEB \times BCDEA.$$

Proposed by Neculai Stanciu – Romania

Solution by Titu Zvonaru-Romania

We denote $p = AC, q = BC, m = DEA, n = DEB$.

Since $ACDEA = 1000 \times AC + DEA = 10^3 p + m$, the desired inequality is equivalent to

$$(10^3 p + m)(10^3 q + n) \leq (10^3 p + n)(10^3 q + m)$$

$$10^6 pq + 10^3 pn + 10^3 qm + mn \leq 10^6 pq + 10^3 pm + 10^3 qn + mn$$

$$pm - pn - qm + qn \geq 0$$

$$(p - q)(m - n) \geq 0 \quad (1)$$

We have $p - q = 10A + C - 10B - C = 10(A - B)$ and

$$m - n = 100D + 10E + A - 100D - 10E - B = A - B.$$

Then the inequality (1) is $10(A - B)^2 \geq 0$, obvious true.