ROMANIAN MATHEMATICAL MAGAZINE

J.2408 If $x,y,z>0, m\geq 1$, then prove that:

$$\sum_{cyc}\frac{x}{mx+y+z}\leq\frac{3}{m+2}$$

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We have:

$$\sum_{cyc}\frac{x}{mx+y+z}-\frac{3}{m+2}=\sum_{cyc}\left(\frac{x}{mx+y+z}-\frac{1}{m+2}\right)=$$

$$=\frac{1}{m+2}\sum_{cyc}\frac{2x-y-z}{mx+y+z}=$$

$$=\frac{1}{m+2}\sum_{cyc}\left(\frac{x-y}{mx+y+z}+\frac{x-z}{mx+y+z}\right)=$$

$$=\frac{1}{m+2}\left(\sum_{cyc}\frac{x-y}{mx+y+z}+\sum_{cyc}\frac{y-x}{x+my+z}\right)=$$

$$=\frac{1}{m+2}\sum_{cyclic}\frac{(x-y)(x+my+z-mx-y-z)}{(mx+y+z)(x+my+z)}=$$

$$=\frac{m-1}{m+2}\sum_{cyclic}\frac{-(x-y)^2}{(mx+y+z)(x+my+z)}\leq 0.$$