ROMANIAN MATHEMATICAL MAGAZINE

J.2412 Prove that 5 divide $n(4n^2+1)(6n^2+1)$, for any natural number n.

Proposed Neculai Stanciu – Romania

Solution by Titu Zvonaru-Romania

For $n=5k$, it is obvious that 5 divide $n(4n^2+1)(6n^2+1)$.

For $n=5k\pm1$, we have $4n^2+1=4(5k\pm1)^2+1=5(20k^2\pm8k+1)$.

For $n=5k\pm2$, we have $6n^2+1=6(5k\pm2)^2+1=5(30k^2\pm24k+5)$.

It follows that 5 divide $n(4n^2+1)(6n^2+1)$.