ROMANIAN MATHEMATICAL MAGAZINE

J.2413 If $M \in Int(\Delta ABC), d_a = d(M, BC), d_b = d(M, CA)$,

$d_c = d(M, AB), F_a = [MBC], F_b = [MCA], F_c = [MAB]$, then:

$$\frac{a^5}{d_a \cdot F_a} + \frac{b^5}{d_b \cdot F_b} + \frac{c^5}{d_c \cdot F_c} \geq 96\sqrt{3} \cdot F$$

Proposed by D.M.Bătinețu-Giurgiu, Claudia Nănuți – Romania

Solution by Titu Zvonaru-Romania

We have $2F_a = ad_a, 2F_b = bd_b, 2F_c = cd_c, ad_a + bd_b + cd_c = 2F$.

Applying Gordon's inequality and Ionescu-Weitzenbock's inequality $a^2 + b^2 + c^2 \geq 4\sqrt{3}F$,

it follows that:

$$\frac{a^5}{d_a \cdot F_a} + \frac{b^5}{d_b \cdot F_b} + \frac{c^5}{d_c \cdot F_c} = \frac{2a^5}{d_a \cdot ad_a} + \frac{2b^5}{d_b \cdot bd_b} + \frac{2c^5}{d_c \cdot cd_c} = \frac{2a^4}{d_a^2} + \frac{2b^4}{d_b^2} + \frac{2c^4}{d_c^2} =$$

$$= 2\left(\frac{(a^2)^3}{(ad_a)^2} + \frac{(b^2)^3}{(bd_b)^2} + \frac{(c^2)^3}{(cd_c)^2}\right) = 2 \cdot \frac{(a^2 + b^2 + c^2)^3}{(ad_a + bd_b + cd_c)^2} \geq 2 \cdot \frac{\left(4\sqrt{3}F\right)^3}{4F^2} = 96\sqrt{3} \cdot F.$$

Equality holds if and only if ΔABC is equilateral and M is the center of the triangle.