ROMANIAN MATHEMATICAL MAGAZINE

J.2415 If $x, y > 0$ then in ΔABC holds:

$$(a^4+b^4+c^4)\left(\frac{1}{xa^2+bcy}+\frac{1}{xb^2+cay}+\frac{1}{xc^2+aby}\right)\ge\frac{12\sqrt{3}}{x+y}F$$

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Using the known inequality $3(x^2+y^2+z^2)\ge(x+y+z)^2$ for a^2,b^2,c^2 (1)

the inequality $ab+bc+ca\le a^2+b^2+c^2$ and Ionescu-Weitzenbock's inequality

$a^2+b^2+c^2\ge 4\sqrt{3}F$, we obtain:

$$(a^4+b^4+c^4)\left(\frac{1}{xa^2+bcy}+\frac{1}{xb^2+cay}+\frac{1}{xc^2+aby}\right)\overset{(1)}{\ge}$$

$$\ge\frac{(a^2+b^2+c^2)^2}{3}\cdot\left(\frac{1}{xa^2+bcy}+\frac{1}{xb^2+cay}+\frac{1}{xc^2+aby}\right)\overset{BERGSTROM}{\ge}$$

$$\ge\frac{(a^2+b^2+c^2)^2}{3}\cdot\frac{9}{x(a^2+b^2+c^2)+y(ab+bc+ca)}\overset{(1)}{\ge}$$

$$\ge\frac{(a^2+b^2+c^2)^2}{3}\cdot\frac{9}{(x+y)(a^2+b^2+c^2)}=$$

$$=\frac{3(a^2+b^2+c^2)}{x+y}\overset{IONESCU-WEITZENBOCK}{\ge}\frac{12\sqrt{3}}{x+y}F$$

Equality holds if and only if ΔABC is equilateral.