ROMANIAN MATHEMATICAL MAGAZINE

J.2418 If $t,u,x,y,z>0$ then:

$$\left(\left(\frac{x}{y+z}+\frac{y}{z+x}\right)^2+t^2\right)\left(\left(\frac{z}{x+y}\right)^2+u^2\right)\ge\left(\frac{ux}{y+z}+\frac{uy}{z+x}+\frac{tz}{x+y}\right)^2$$

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Applying Cauchy-Buniakovski-Schwarz inequality, it follows that

$$\left(\left(\frac{x}{y+z}+\frac{y}{z+x}\right)^2+t^2\right)\left(\left(\frac{z}{x+y}\right)^2+u^2\right)=\left(\left(\frac{x}{y+z}+\frac{y}{z+x}\right)^2+t^2\right)\left(u^2+\left(\frac{z}{x+y}\right)^2\right)$$

$$\ge\left(u\left(\frac{ux}{y+z}+\frac{uy}{z+x}\right)+\frac{tz}{x+y}\right)^2=\left(\frac{ux}{y+z}+\frac{uy}{z+x}+\frac{tz}{x+y}\right)^2.$$

Equality holds if and only if $\left(\frac{x}{y+z}+\frac{y}{z+x}\right)\left(\frac{z}{x+y}\right)=tu.$