ROMANIAN MATHEMATICAL MAGAZINE

J.2424 Find all $(m,n,p)\in N\times N\times N$ such that:

$$m+n+p+mn+np+pm+mnp=35$$
$$(p+2)\cdot p^p=(p+2)^p$$

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Solution by Titu Zvonaru-Romania

Writing the second equation as $p+2=\left(1+\frac{2}{p}\right)^p$,

we deduce $1+\frac{2}{p}$ must be a natural number.

We get $p\in\{1,2\}$. The first equation is equivalent to

$$1+m+n+p+mn+np+pm+mnp=36$$
$$m+1+n(m+1)+p(m+1)+np(m+1)=36$$
$$(m+1)(n+1)(p+1)=36.$$

For $p=1$ yields $(m+1)(n+1)=18$, hence

$$(m,n,p)=(0,17,1),(1,8,1),(2,5,1),(5,2,1),(8,1,1),(17,0,1)$$

For $p=2$ yields $(m+1)(n+1)=12$, hence

$$(m,n,p)=(0,11,2),(1,5,2),(2,3,2),(3,2,2),(5,1,2),(11,0,2)$$