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J.2433 In $\triangle ABC$ the following relationship holds

$$6r\min\{a, b, c\} \leq 2Rs \leq 3R\max\{a, b, c\}$$

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We have $a, b, c \geq \min\{a, b, c\}$ and $a, b, c \leq \max\{a, b, c\}$.

Using Euler inequality $R \geq 2r$, we obtain

$$2Rs = R(a + b + c) \geq R(\min\{a, b, c\} + \min\{a, b, c\} + \min\{a, b, c\}) \geq 6r\min\{a, b, c\}$$

$$\text{and } 2Rs = R(a + b + c) \leq R(\max\{a, b, c\} + \max\{a, b, c\} + \max\{a, b, c\}) \leq \\ \leq 3R\max\{a, b, c\}.$$

Equality holds if and only if $\triangle ABC$ is equilateral.