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J.2457 If $m, n \geq 0$ then in ΔABC holds:

$$\frac{m^3a^3+n^3b^3}{ab}+\frac{m^3b^3+n^3c^3}{bc}+\frac{m^3c^3+n^3a^3}{ca}\geq\frac{(m+n)^3s}{2}$$

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By Hölder's inequality we obtain

$$4(m^3+n^3)=(1+1)(1+1)(m^3+n^3)\geq(m+n)^3 \qquad (1)$$

Applying Bergström inequality and (1), it follows that

$$m^3\left(\frac{a^3}{ab}+\frac{b^3}{bc}+\frac{c^3}{ca}\right)+n^3\left(\frac{b^3}{ab}+\frac{c^3}{bc}+\frac{a^3}{ca}\right)=m^3\left(\frac{a^2}{b}+\frac{b^2}{c}+\frac{c^2}{a}\right)+n^3\left(\frac{b^2}{a}+\frac{c^3}{b}+\frac{a^3}{c}\right)\geq$$

$$\geq m^3\cdot\frac{(a+b+c)^2}{b+c+a}+n^3\cdot\frac{(b+c+a)^2}{a+b+c}=2(m^3+n^3)s\geq\frac{(m+n)^3s}{2}.$$

Equality holds if and only if ΔABC is equilateral and $m=n$.