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J.2461 In acute ΔABC holds:

$$\sum_{cyc}\frac{\sec^2 B+\sec^2 C}{\sec A}\ge 12$$

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Applying Bergström's inequality and the inequality

$\sec A+\sec B+\sec C\ge 6$ (item 2.45 from $[1]$), it follows that:

$$\sum_{cyc}\frac{\sec^2 B+\sec^2 C}{\sec A}=\sum_{cyc}\frac{\sec^2 B}{\sec A}+\sum_{cyc}\frac{\sec^2 C}{\sec A}\ge$$

$$\ge\frac{(\sec B+\sec C+\sec A)^2}{\sec A+\sec B+\sec C}+\frac{(\sec C+\sec A+\sec B)^2}{\sec A+\sec B+\sec C}=2(\sec A+\sec B+\sec C)\ge 12.$$

Equality holds if and only if ΔABC is equilateral.

[1] O. Bottema, *Geometric Inequalities*, Groningen 1969