ROMANIAN MATHEMATICAL MAGAZINE

J.2463 Let $f(x)=ax^2+bx+c, g(x)=cx^2+ax+b$ $(a,b,c\in R)$. Find all values a,b,c such that $f(g(x))=x^4-2x^3+4x^2-3x+1, x\in R$.

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We have:

$$f(g(x))=a(cx^2+ax+b)^2+b(cx^2+ax+b)+c=$$

$$=ac^2x^4+2a^2cx^3+(a^3+2abc+bc)x^2+(2a^2b+ab)x+ab^2+b^2+c.$$

It follows that

$$ac^2=1, 2a^2c=-2, a^3+2abc+bc=4, 2a^2b+ab=-3, ab^2+b^2+c=1.$$

By $ac^2=1, ac^2=-1$ we get $a^3c^3=-1$, hence $ac=-1, a=1, c=-1$.

It remains: $1-2b-b=4, 2b+b=-3, b^2+b^2-1=1 \Rightarrow b=-1$.