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J.2470 If $x,y\geq 0$ then in ΔABC holds:

$$a^xb^y+b^xc^y+c^xa^y\geq 2^{x+y}3^{\frac{4-x-y}{4}}\left(\sqrt{F}\right)^{x+y}$$

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Applying $AM-GM$ inequality and Carlitz's inequality $(abc)^{2/3}\geq\frac{4}{\sqrt{3}}F$

(item 4.14 from $[1]$) it follows that:

$$a^xb^y+b^xc^y+c^xa^y\geq 3\left(a^{x+y}b^{x+y}c^{x+y}\right)^{\frac{1}{3}}=$$

$$=3\left((abc)^{\frac{2}{3}}\right)^{\frac{x+y}{2}}\geq 3\left(\frac{4}{\sqrt{3}}F\right)^{\frac{x+y}{2}}=2^{x+y}3^{\frac{4-x-y}{4}}\left(\sqrt{F}\right)^{x+y}.$$

Equality holds if and only if ΔABC is equilateral.

$[1]$ O. Bottema, *Geometric Inequalities*, Groningen 1969