J.2471 In $\triangle ABC$ the following relationship holds

$$\frac{m_a}{h_a}bc + \frac{m_b}{h_b}ca + \frac{m_c}{h_c}ab \geq 4\sqrt{3}F$$

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Since $m_a \geq h_a$, using Gordon's inequality $bc + ca + ab \geq 4\sqrt{3}F$ we obtain:

$$\frac{m_a}{h_a}bc + \frac{m_b}{h_b}ca + \frac{m_c}{h_c}ab \geq ab + bc + ca \geq 4\sqrt{3}F.$$

Equality holds if and only if $\triangle ABC$ is equilateral.