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J.2474 If $x, y, z > 0$ then in triangle ABC holds:

$$\frac{x}{\sqrt{yz}}\cdot a+\frac{y}{\sqrt{zx}}\cdot b+\frac{z}{\sqrt{xy}}\cdot c\ge 2\cdot 3^{\frac{3}{4}}\cdot\sqrt{F}$$

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Applying $AM-GM$ inequality and Carlitz's inequality $(abc)^{2/3}\ge\frac{4}{\sqrt{3}}F$,

it follows that:

$$\frac{x}{\sqrt{yz}}\cdot a+\frac{y}{\sqrt{zx}}\cdot b+\frac{z}{\sqrt{xy}}\cdot c\ge 3\left(\frac{x}{\sqrt{yz}}\cdot a\cdot\frac{y}{\sqrt{zx}}\cdot b\cdot\frac{z}{\sqrt{xy}}\cdot c\right)^{\frac{1}{3}}=$$

$$=3(abc)^{\frac{1}{3}}\ge 3\left(\frac{4}{\sqrt{3}}F\right)^{\frac{1}{2}}=2\cdot 3^{\frac{3}{4}}\cdot\sqrt{F}.$$

Equality holds if and only if triangle ABC and $x=y=z$.